CS188 Fall 2018 Section 6: Probability + Bayes' Nets

1 Probability

Use the probability table to calculate the following values:

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

1. $P(X_1 = 1, X_2 = 0) = 0.15$

2. $P(X_3 = 0) = 0.65$

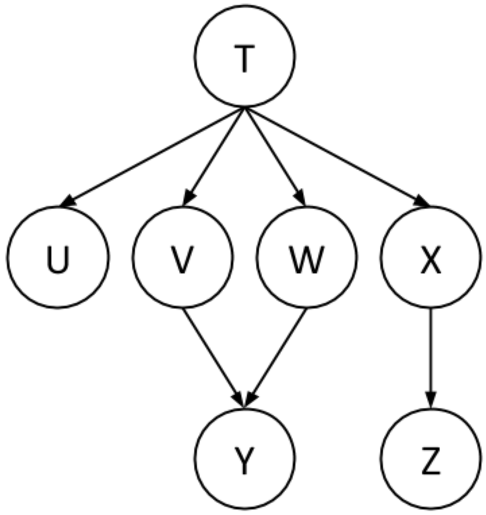
3. $P(X_2 = 1 | X_3 = 1) = 0.2/0.35$

4. $P(X_1 = 0 | X_2 = 1, X_3 = 1) = 1$

5. $P(X_1 = 0, X_2 = 1 | X_3 = 1) = 0.2/0.35$

2 D-Separation

Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph.



1. $U \perp\!\!\!\perp X$

   Not guaranteed, path U-T-X is active

2. $U \perp\!\!\!\perp X|T$

   Guaranteed

3. $V \perp\!\!\!\perp W|Y$

   Not guaranteed, paths V-T-W and V-Y-W are both active

4. $V \perp\!\!\!\perp W|T$

   Guaranteed

5. $T \perp\!\!\!\perp Y|V$

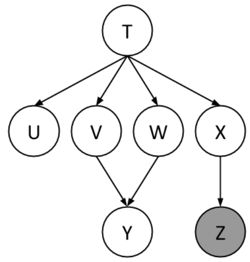
   Not guaranteed, path T-W-Y is active

6. $Y \perp\!\!\!\perp Z|W$

   Not guaranteed, path Y-V-T-X-Z is active

7. $Y \perp\!\!\!\perp Z|T$

   Guaranteed, with T being observed, there are no active paths from Y to Z



3 Variable Elimination

Using the same Bayes Net (shown below), we want to compute $P(Y \mid +z)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: X, T, U, V, W.

Complete the following description of the factors generated in this process:

After inserting evidence, we have the following factors to start out with:

$$P(T), P(U|T), P(V|T), P(W|T), P(X|T), P(Y|V,W), P(+z|X)$$

(a) When eliminating X we generate a new factor f_1 as follows, which leaves us with the factors:

$$f_1(+z|T) = \sum_x P(x|T)P(+z|x) \qquad P(T), P(U|T), P(V|T), P(W|T), P(Y|V,W), f_1(+z|T)$$

(b) When eliminating T we generate a new factor f_2 as follows, which leaves us with the factors:

$$f_2(U,V,W,+z) = \sum_t P(t)P(U|t)P(V|t)P(W|t)f_1(+z|t) \qquad P(Y|V,W), f_2(U,V,W,+z)$$

(c) When eliminating U we generate a new factor f_3 as follows, which leaves us with the factors:

$$f_3(V,W,+z) = \sum_u f_2(u,V,W,+z) \qquad P(Y|V,W), f_3(V,W,+z)$$

(d) When eliminating V we generate a new factor f_4 as follows, which leaves us with the factors:

$$f_4(W,Y,+z) = \sum_v f_3(v,W,+z)P(Y|v,W) \qquad f_4(W,Y,+z)$$

(e) When eliminating W we generate a new factor f_5 as follows, which leaves us with the factors:

$$f_5(Y, +z) = \sum_w f_4(w, Y, +z) \qquad f_5(Y, +z)$$

(f) How would you obtain $P(Y \mid +z)$ from the factors left above:
Simply renormalize $f_5(Y, +z)$ to obtain $P(Y \mid +z)$. Concretely,

$$P(y \mid +z) = \frac{f_5(y, +z)}{\sum_{y'} f_5(y', +z)}$$

(g) What is the size of the largest factor that gets generated during the above process?
$f_2(U, V, W, +z)$. This contains 3 unconditioned variables, so it will have $2^3 = 8$ probability entries (U, V, W are binary variables, and we only need to store the probability for $+z$ for each possible setting of these variables).

(m) Does there exist a better elimination ordering (one which generates smaller largest factors)?
Yes. One such ordering is X, U, T, V, W. All factors generated with this ordering contain at most 2 unconditioned variables, so the tables will have at most $2^2 = 4$ probability entries (as all variables are binary).