CS 188
Fall 2018

Introduction to
Artificial Intelligence

Written HW 6 Sol.

Self-assessment due: Monday 10/22/2018 at 11:59pm (submit via Gradescope)

For the self assessment, **fill in the self assessment boxes in your original submission** (you can download a PDF copy of your submission from Gradescope – be sure to delete any extra title pages that Gradescope attaches). For each subpart where your original answer was correct, write "correct." Otherwise, write and explain the correct answer. **Do not leave any boxes empty.** If you did not submit the homework (or skipped some questions) but wish to receive credit for the self-assessment, we ask that you first complete the homework without looking at the solutions, and then perform the self assessment afterwards.

Q1. Probability

(a) For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, mark "Not possible."

(i) Using probability tables $\mathbf{P}(\mathbf{A}), \mathbf{P}(\mathbf{A} \mid \mathbf{C}), \mathbf{P}(\mathbf{B} \mid \mathbf{C}), \mathbf{P}(\mathbf{C} \mid \mathbf{A}, \mathbf{B})$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P}(\mathbf{A}, \mathbf{B} \mid \mathbf{C})$.

$\mathbf{P}(\mathbf{A}, \mathbf{B} \mid \mathbf{C}) =$ ________________ ● Not possible.

(ii) Using probability tables $\mathbf{P}(\mathbf{A}), \mathbf{P}(\mathbf{A} \mid \mathbf{C}), \mathbf{P}(\mathbf{B} \mid \mathbf{A}), \mathbf{P}(\mathbf{C} \mid \mathbf{A}, \mathbf{B})$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C})$.

$\mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C}) = \dfrac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_b P(A)\ P(B|A)\ P(C|A,B)}$ ○ Not possible.

(iii) Using probability tables $\mathbf{P}(\mathbf{A} \mid \mathbf{B}), \mathbf{P}(\mathbf{B}), \mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C}), \mathbf{P}(\mathbf{C} \mid \mathbf{A})$ and conditional independence assumption $\mathbf{A} \perp\!\!\!\perp \mathbf{B}$, write an expression to calculate the table $\mathbf{P}(\mathbf{C})$.

$\mathbf{P}(\mathbf{C}) = \sum_a P(A \mid B)\ P(C \mid A)$ ○ Not possible.

(iv) Using probability tables $\mathbf{P}(\mathbf{A} \mid \mathbf{B}, \mathbf{C}), \mathbf{P}(\mathbf{B}), \mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C}), \mathbf{P}(\mathbf{C} \mid \mathbf{B}, \mathbf{A})$ and conditional independence assumption $\mathbf{A} \perp\!\!\!\perp \mathbf{B} \mid \mathbf{C}$, write an expression for $\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C})$.

$\mathbf{P}(\mathbf{A}, \mathbf{B}, \mathbf{C}) =$ ________________ ● Not possible.

(b) For each of the following equations, select the *minimal set* of conditional independence assumptions necessary for the equation to be true.

(i) $\mathbf{P}(\mathbf{A}, \mathbf{C}) = \mathbf{P}(\mathbf{A} \mid \mathbf{B})\ \mathbf{P}(\mathbf{C})$

- ■ $A \perp\!\!\!\perp B$
- □ $A \perp\!\!\!\perp B \mid C$
- ■ $A \perp\!\!\!\perp C$
- □ $A \perp\!\!\!\perp C \mid B$
- □ $B \perp\!\!\!\perp C$
- □ $B \perp\!\!\!\perp C \mid A$
- □ No independence assumptions needed.

(ii) $\mathbf{P}(\mathbf{A} \mid \mathbf{B}, \mathbf{C}) = \dfrac{\mathbf{P}(\mathbf{A})\ \mathbf{P}(\mathbf{B}|\mathbf{A})\ \mathbf{P}(\mathbf{C}|\mathbf{A})}{\mathbf{P}(\mathbf{B}|\mathbf{C})\ \mathbf{P}(\mathbf{C})}$

- □ $A \perp\!\!\!\perp B$
- □ $A \perp\!\!\!\perp B \mid C$
- □ $A \perp\!\!\!\perp C$
- □ $A \perp\!\!\!\perp C \mid B$
- □ $B \perp\!\!\!\perp C$
- ■ $B \perp\!\!\!\perp C \mid A$
- □ No independence assumptions needed.

(iii) $\mathbf{P}(\mathbf{A}, \mathbf{B}) = \sum_{\mathbf{c}} \mathbf{P}(\mathbf{A} \mid \mathbf{B}, \mathbf{c})\ \mathbf{P}(\mathbf{B} \mid \mathbf{c})\ \mathbf{P}(\mathbf{c})$

- □ $A \perp\!\!\!\perp B$
- □ $A \perp\!\!\!\perp B \mid C$
- □ $A \perp\!\!\!\perp C$
- □ $A \perp\!\!\!\perp C \mid B$
- □ $B \perp\!\!\!\perp C$
- □ $B \perp\!\!\!\perp C \mid A$
- ■ No independence assumptions needed.

(iv) $\mathbf{P}(\mathbf{A}, \mathbf{B} \mid \mathbf{C}, \mathbf{D}) = \mathbf{P}(\mathbf{A} \mid \mathbf{C}, \mathbf{D})\ \mathbf{P}(\mathbf{B} \mid \mathbf{A}, \mathbf{C}, \mathbf{D})$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp B \mid D$
- ☐ $C \perp\!\!\!\perp D$
- ☐ $C \perp\!\!\!\perp D \mid A$
- ☐ $C \perp\!\!\!\perp D \mid B$
- ■ No independence assumptions needed.

(c) **(i)** Mark **all** expressions that are equal to $\mathbf{P(A \mid B)}$, **given no independence assumptions**.

- ☐ $\sum_c P(A \mid B, c)$
- ■ $\sum_c P(A, c \mid B)$
- ☐ $\frac{P(B|A)\ P(A|C)}{\sum_c P(B,c)}$
- ■ $\frac{\sum_c P(A,B,c)}{\sum_c P(B,c)}$
- ☐ $\frac{P(A,C|B)}{P(C|B)}$
- ☐ $\frac{P(A|C,B)\ P(C|A,B)}{P(C|B)}$
- ☐ None of the provided options.

(ii) Mark **all** expressions that are equal to $\mathbf{P(A, B, C)}$, given that $\mathbf{A} \perp\!\!\!\perp \mathbf{B}$.

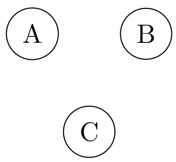
- ☐ $P(A \mid C)\ P(C \mid B)\ P(B)$
- ■ $P(A)\ P(B)\ P(C \mid A, B)$
- ☐ $P(C)\ P(A \mid C)\ P(B \mid C)$
- ☐ $P(A)\ P(C \mid A)\ P(B \mid C)$
- ■ $P(A)\ P(B \mid A)\ P(C \mid A, B)$
- ■ $P(A, C)\ P(B \mid A, C)$
- ☐ None of the provided options.

(iii) Mark **all** expressions that are equal to $\mathbf{P(A, B \mid C)}$, given that $\mathbf{A} \perp\!\!\!\perp \mathbf{B} \mid \mathbf{C}$.

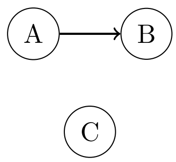
- ■ $P(A \mid C)\ P(B \mid C)$
- ☐ $\frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_c P(A,B,c)}$
- ☐ $P(A \mid B)\ P(B \mid C)$
- ☐ $\frac{P(C)\ P(B|C)\ P(A|C)}{P(C|A,B)}$
- ☐ $\frac{\sum_c P(A,B,c)}{P(C)}$
- ■ $\frac{P(C,A|B)\ P(B)}{P(C)}$
- ☐ None of the provided options.

Q2. Bayes' Nets: Representation

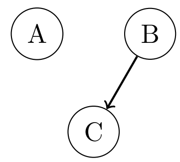
Assume we are given the following ten Bayes' nets, labeled $\mathbf{G_1}$ to $\mathbf{G_{10}}$:



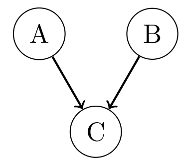
$\mathbf{G_1}$



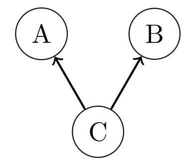
$\mathbf{G_2}$



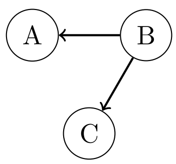
$\mathbf{G_3}$



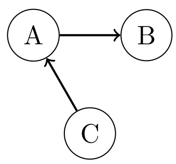
$\mathbf{G_4}$



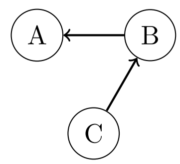
$\mathbf{G_5}$



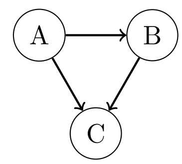
$\mathbf{G_6}$



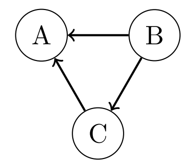
$\mathbf{G_7}$



$\mathbf{G_8}$

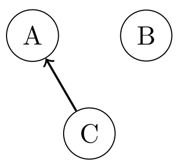


$\mathbf{G_9}$

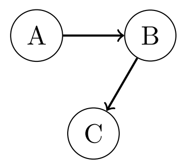


$\mathbf{G_{10}}$

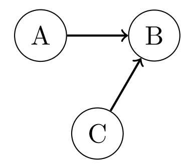
Assume we are also given the following three Bayes' nets, labeled $\mathbf{B_1}$ to $\mathbf{B_3}$:



$\mathbf{B_1}$



$\mathbf{B_2}$



$\mathbf{B_3}$

Before we go into the questions, let's enumerate all of the (conditional) independence assumptions encoded in all the Bayes' nets above. They are:

- $\mathbf{G_1}$: $AB; AB|C; AC; AC|B; BC; BC|A$
- $\mathbf{G_2}$: $AC; AC|B; BC; BC|A$
- $\mathbf{G_3}$: $AB; AB|C; AC; AC|B$
- $\mathbf{G_4}$: AB
- $\mathbf{G_5}$: $AB|C$
- $\mathbf{G_6}$: $AC|B$
- $\mathbf{G_7}$: $BC|A$
- $\mathbf{G_8}$: $AC|B$
- $\mathbf{G_9}$: $\emptyset$
- $\mathbf{G_{10}}$: $\emptyset$
- $\mathbf{B_1}$: $AB; AB|C; BC; BC|A$
- $\mathbf{B_2}$: $AC|B$
- $\mathbf{B_3}$: AC

(a) Assume we know that a joint distribution $\mathbf{d_1}$ (over $\mathbf{A}, \mathbf{B}, \mathbf{C}$) can be represented by Bayes' net $\mathbf{B_1}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_1}$.

☐ $\mathbf{G_1}$ ☐ $\mathbf{G_2}$ ☐ $\mathbf{G_3}$ ■ $\mathbf{G_4}$ ■ $\mathbf{G_5}$

☐ $\mathbf{G_6}$ ■ $\mathbf{G_7}$ ☐ $\mathbf{G_8}$ ■ $\mathbf{G_9}$ ■ $\mathbf{G_{10}}$

☐ None of the above.

Since $\mathbf{B_1}$ can represent $\mathbf{d_1}$, we know that $\mathbf{d_1}$ must satisfy the assumptions that $\mathbf{B_1}$ follows, which are: $AB; AB|C; BC; BC|A$. We cannot assume that $\mathbf{d_1}$ satisfies the other two assumptions, which are AC and $AC|B$, and so a Bayes' net that makes at least one of these two extra assumptions will not be guaranteed to be able to represent $\mathbf{d_1}$. This eliminates the choices $\mathbf{G_1, G_2, G_3, G_6, G_8}$. The other choices $\mathbf{G_4, G_5, G_7, G_9, G_{10}}$ are guaranteed to be able to represent $\mathbf{d_1}$ because they do not make any additional independence assumptions that $\mathbf{B_1}$ makes.

(b) Assume we know that a joint distribution $\mathbf{d_2}$ (over $\mathbf{A, B, C}$) can be represented by Bayes' net $\mathbf{B_2}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_2}$.

☐ $\mathbf{G_1}$	☐ $\mathbf{G_2}$	☐ $\mathbf{G_3}$	☐ $\mathbf{G_4}$	☐ $\mathbf{G_5}$
■ $\mathbf{G_6}$	☐ $\mathbf{G_7}$	■ $\mathbf{G_8}$	■ $\mathbf{G_9}$	■ $\mathbf{G_{10}}$

☐ None of the above.

Since $\mathbf{B_2}$ can represent $\mathbf{d_2}$, we know that $\mathbf{d_2}$ must satisfy the assumptions that $\mathbf{B_2}$ follows, which is just: $AC|B$. We cannot assume that $\mathbf{d_2}$ satisfies any other assumptions, and so a Bayes' net that makes at least one other extra assumptions will not be guaranteed to be able to represent $\mathbf{d_2}$. This eliminates the choices $\mathbf{G_1, G_2, G_3, G_4, G_5, G_7}$. The other choices $\mathbf{G_6, G_8, G_9, G_{10}}$ are guaranteed to be able to represent $\mathbf{d_2}$ because they do not make any additional independence assumptions that $\mathbf{B_2}$ makes.

(c) Assume we know that a joint distribution $\mathbf{d_3}$ (over $\mathbf{A, B, C}$) ***cannot*** be represented by Bayes' net $\mathbf{B_3}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_3}$.

☐ $\mathbf{G_1}$	☐ $\mathbf{G_2}$	☐ $\mathbf{G_3}$	☐ $\mathbf{G_4}$	☐ $\mathbf{G_5}$
☐ $\mathbf{G_6}$	☐ $\mathbf{G_7}$	☐ $\mathbf{G_8}$	■ $\mathbf{G_9}$	■ $\mathbf{G_{10}}$

☐ None of the above.

Since $\mathbf{B_3}$ cannot represent $\mathbf{d_3}$, we know that $\mathbf{d_3}$ is unable to satisfy at least one of the assumptions that $\mathbf{B_3}$ follows. Since $\mathbf{B_3}$ only makes one independence assumption, which is AC, we know that $\mathbf{d_3}$ does not satisfy AC. However, we can't claim anything about whether or not $\mathbf{d_3}$ makes any of the other independence assumptions. $\mathbf{d_3}$ might not make any (conditional) independence assumptions at all, and so only the Bayes' nets that don't make any assumptions will be guaranteed to be able to represent $\mathbf{d_3}$. Hence, the answers are the fully connected Bayes' nets, which are $\mathbf{G_9, G_{10}}$.

(d) Assume we know that a joint distribution $\mathbf{d_4}$ (over $\mathbf{A, B, C}$) can be represented by Bayes' nets $\mathbf{B_1}$, $\mathbf{B_2}$, and $\mathbf{B_3}$. Mark all of the following Bayes' nets that are guaranteed to be able to represent $\mathbf{d_4}$.

■ $\mathbf{G_1}$	■ $\mathbf{G_2}$	■ $\mathbf{G_3}$	■ $\mathbf{G_4}$	■ $\mathbf{G_5}$
■ $\mathbf{G_6}$	■ $\mathbf{G_7}$	■ $\mathbf{G_8}$	■ $\mathbf{G_9}$	■ $\mathbf{G_{10}}$

☐ None of the above.

Since $\mathbf{B_1, B_2, B_3}$ can represent $\mathbf{d_4}$, we know that $\mathbf{d_4}$ must satisfy the assumptions that $\mathbf{B_1, B_2, B_3}$ make. The union of assumptions made by these Bayes' nets are: $AB; AB|C; BC; BC|A, AC, AC|B$. Note that this set of assumptions encompasses all the possible assumptions that you can make with 3 random variables, so any Bayes' net over $\mathbf{A, B, C}$ will be able to represent $\mathbf{d_4}$.